Raj Kumar Goel Institute of Technology and Management, Ghaziabad

First Sessional Examination

Applied Linear Algebra ROE-039

Max Marks:50

Note: Attempt All Sections: **Time: 90 min**

Section-A

Attempt all questions: **(2*5=10)**

(1). Define field with example.

(2). For what value of m ,the vector (m,3,1)is a linear combination of vectors (3,2,1)and (2,1,0).

(3) . Define linear transformation.

(4). Prove that a set consisting of a single non zero vector is always linearly independent.

(5). Define dimension of a vector space.

Section-B

Attempt any five question: **(5*5=25)**

(1). Show that the set W of the elements of the vector space $V_3(R)$ of the form (x+2y,y,-x+3y) where $x,y \in R$ is a subspace of $V_3(R)$.

(2). Show that the system of three vector (1,3,2),(1,-7,-8),(2,1,-1) of $V_3(R)$ is linearly dependent.

(3). Determine whether or not the following vector form a basis of R^3,(1,1,2),(1,2,5),(5,3,4).

(4) Find a rank of the system of vectors e_1=(2,-2,-4,), e_2=(1,9,3), e_3=(-2,-4,1),), e_4=(3,7,-1).

(5). Consider the basis S={e_1,e_2,e_3} of R^3 where e_1=(1,1,1,), e_2=(1,1,0), e_3=(1,0,0), express (2,-3,5) in terms of basis e_1,e_2,e_3.

(6) Find the coordinates of the vector (2,1,-6)of R^3relative to the basis where e_1=(1,1,2,), e_2=(3,-1,0), e_3=(2,0,-1).

(7) Show that the set W={(xy,y): $xy \geq 0$}of the vector space $V_2(R)$ where $x,y \in R$ is not a subspace of $V_2(R)$.

(8) Show that the mapping $T:R^2 \rightarrow R$ defined by T(x,y)=xy is not linear .

Section-C

Attempt any two questions: **(7.5*2=15)**

(1). Prove that all polynomials over a field F is a vector space.

(2). Let T be the linear operator on R^3 defined by $T(x_1,x_2,x_3)=(3x_1+x_3,-2x_1+x_2,-x_1+2x_2+4x_3)$. What is the matrix of T in the basis ={e_1,e_2,e_3} where e_1=(1,0,1),e_2=(-1,2,1) and e_3=(2,1,1) and usual basis $e_1^{'}$=(1,0,0),$e_2^{'}$=(-0,1,0) and $e_3^{'}$=(0,0,1)

(3). Show that the mapping $T:V_2(R)\rightarrow V_3(R)$ defined as T(a,b)=(a+b,a-b,b) is linear transformation from V_2(R) into V_3(R). Find the range , rank,null space and nullity of T.